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**SUPPLEMENTARY NORMAL
ACCELERATION a_n^***

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ORDIS EDITIONS

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SUPPLEMENTARY NORMAL ACCELERATION a_n^*

KINEMATIC AND DYNAMIC MEANING OF ANGULAR VELOCITY $\dot{\varphi}^*$

1. At first, we begin with the study of the trajectory of a material point from the kinematic point of view exclusively. In classical kinematics a differential ds of arc in the trajectory is substituted by the corresponding in the osculating circle in order to calculate the acceleration vector. For this purpose a FRENET's referential frame is used. The acceleration components in this circle are

$$\mathbf{a}_s = (dv/dt)\mathbf{s} \quad \text{and} \quad \mathbf{a}_\varphi = -(v^2/\rho)\mathbf{n} \quad (1)$$

And where \mathbf{s} and \mathbf{n} are the *versors*, in this frame: \mathbf{s} , \mathbf{n} , \mathbf{b} , whose positive sense in the *tangent*, *normal* and *binormal*, is determined by the velocity sense, by the sense towards convexity and by the vectorial product: $\mathbf{b} = \mathbf{s} \wedge \mathbf{n}$, respectively. The angular velocity is

$$\dot{\varphi} = (v/\rho)\mathbf{b}$$

A definite trajectory has a well defined *evolute*, and in the calculation of the normal component in the expressions (1) the differentials dv and $d\rho$ are obviously not taken into account. But, as we will demonstrate, when $dv \neq 0$ and $d\rho \neq 0$, the *arc* of the *evolute* does not correspond with the real one: it turns locally at an angular velocity

$$\dot{\varphi} = (dv/d\rho)\mathbf{b}$$

and the same thing occurs with the corresponding *arc of trajectory* in the osculating circle.

In order to explain the kinematic meaning of this angular velocity $\dot{\varphi}$, we shall study an element ds of trajectory which corresponds to the $d\varphi$ of the *evolute*; they are both located on the plane of osculation (see Fig.1 when $dv/dt > 0$; and Fig. 2 when $dv/dt < 0$). Thus we can consider the trajectory as being locally plane and referred to an intrinsic frame with *versors* \mathbf{s} , \mathbf{n} , \mathbf{b} , formed by the *tangent*, *normal* and the *binormal*. The *arc* ds of the trajectory is determined by the points A, B , and the $d\varphi$, of the *evolute*, on account of its equivalent points A, B . The speed of the particle in A is v and in B it is $v + dv$. The radii of curvature at these points are: $\varrho + d\varrho$ and ϱ . The angle turned by the radius of curvature when it passes from A to B is

$$d\varphi = ds/\varrho$$

and the corresponding angular speed will be as we have seen

$$\dot{\varphi} = d\varphi/dt \quad (\text{with } \dot{\varphi} = \dot{\varphi} \mathbf{b})$$

We can also write: $\dot{\varphi} = v/\varrho$, which evidently does not depend on dv and $d\varrho$. When we calculate the centripetal acceleration we get the expression (1) in the form

$$\mathbf{a}_{\varrho} = (-v^2/\varrho)\mathbf{n}$$

in which the increases dv , $d\varrho$, are not considered, as they do not affect it. It is the consequence of replacing the ds of *trajectory* by corresponding one in the *osculating circle* at the same point. However, if we observe the real trajectory carefully, we see that is characterized by having a well determined *evolute* (see Fig. 1, when $dv/dt > 0$, and Fig. 2, when $dv/dt < 0$). When dv is dispensed with, in the study of centripetal acceleration, it

means that starting out from point A we arrive at B' but not at the real point B ; and the same must also be said of its equivalent centre of curvature: A is located in the *evolute*, as it is the starting point, but A' lay outside of the *real evolute* (see Fig.1 and 2), whose point is B . It is evident that the centripetal acceleration is correctly determined, but it is also clear that the *arc* of the *evolute* must coincide with what is determined by points A and B in the figure, and not by the A and B' , as happens when dv and $d\varphi$ are omitted. In order to rectify this deficiency it is necessary to rotate AB' an angle

$$d\varphi^* = BB'/d\varphi$$

so that it coincides with the $d\varphi$ in the *evolute*, with a *finite* angular velocity (see Fig. 1 and Fig. 2) whose module is expressed by

$$(BB'/d\varphi)/dt = (d^2s/d\varphi^2)/dt = dv/d\varphi = d\varphi^*/dt = \varphi^*$$

This angular velocity shows that the simplification of replacing the trajectory with the osculating circle in each point means that it is necessary to turn locally the *arc* of the *evolute*, with angular velocity φ^* , so that it coincides with the real one. But this *arc* AB' of the *evolute* must be *normal* to the corresponding AB'' of the *trajectory*, rotated also $d\varphi^*$, with respect to the initial AB (see Fig. 1 and Fig. 2). It will be necessary to turn AB' this angle, in the *same sense* (when $dv/dt > 0$) and in the *opposite sense* (when $dv/dt < 0$), so that it coincides with the *real* one. As a result, the *radius* φ has increased in a second order infinitesimal amount:

$$B'B'' = dsd\varphi^2 \quad (\text{when } dv/dt > 0)$$

and

$$B'B'' = -dsd\varphi^2 \quad (\text{when } dv/dt < 0)$$

and the immediate result is a *supplementary normal acceleration*:

$$\begin{aligned} a_{\square}^* &= B'B''/dt^2 = ds d^2\square^*/dt^2 = v\square^* \quad (\text{when } dv/dt > 0) \\ a_{\square}^* &= B'B''/dt^2 = -ds d^2\square^*/dt^2 = -v\square^* \quad (\text{when } dv/dt < 0) \end{aligned}$$

superimposed to the *normal acceleration* a_{\square} (1). So the *total normal acceleration* is

$$\begin{aligned} a_{\square} + a_{\square}^* &= -(v\square + v\square^*) = -v(\square - \square^*) \\ a_{\square} + a_{\square}^* &= -(v\square - v\square^*) = -v(\square + \square^*) \end{aligned}$$

in the two possible cases.

Obviously the *tangential acceleration* $a_s = dv/dt$ remains unchanged. In vectorial form we get

$$\begin{aligned} a_s \mathbf{s} + a_{\square} \mathbf{n} + a_{\square}^* \mathbf{n} &= \mathbf{a} + v \square \mathbf{n} = \mathbf{a} - v \square \mathbf{n} \\ a_s \mathbf{s} + a_{\square} \mathbf{n} + a_{\square}^* \mathbf{n} &= \mathbf{a} - v \square \mathbf{n} = \mathbf{a} + v \square \mathbf{n} \end{aligned} \tag{2}$$

respectively.

2. Now, from the dynamical point of view, if we want to calculate the *total centripetal force* correctly, the *total normal acceleration* (16) must be taken into account. So the expression of this force will be

$$\begin{aligned} \mathbf{f}_n &= -mv (\square - \square \mathbf{n}) = mv \square (\square - \mathbf{n}) \\ \mathbf{f}_n &= -mv (\square + \square \mathbf{n}) = mv \square (\square + \mathbf{n}) \end{aligned} \tag{3}$$

and

in both possible cases.

In summary, taken in account the expression (2), the *total force* acting on the material point is

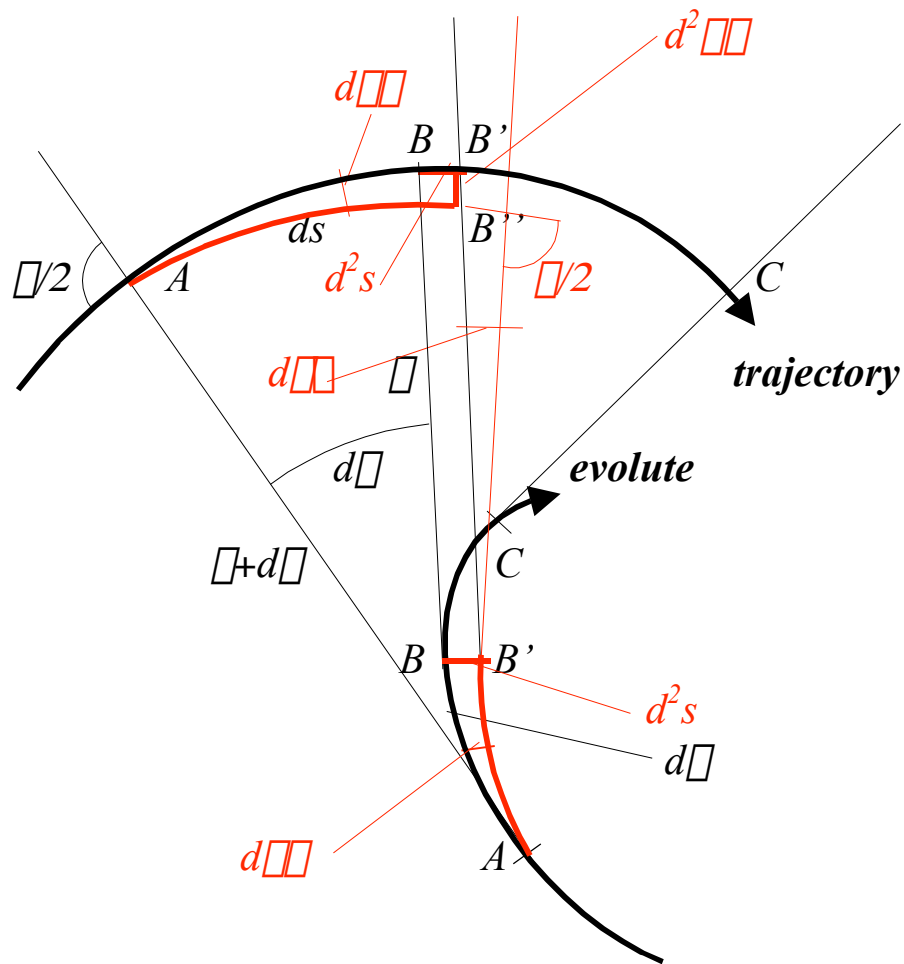
$$\mathbf{f} = m(\mathbf{a} \pm \mathbf{v} \frac{d\mathbf{v}}{ds}) \quad (4)$$

(which is *isomorphic* with the LORENTZ electromagnetic force).

The angular velocity $\frac{d\mathbf{v}}{ds}$ will only cease to exist when the trajectory is a circumference or the speed v is constant, as it follows observing Fig.1 and Fig. 2 .(see also the cases of Fig 1' and Fig.2').

The result (4) is surprising: even more so when we remember that "LORENTZ's force" is exclusively experimental. Moreover, in FRENET's trihedron the value v of speed is always *positive* in the sense in which the particle is moving . We know that while the moving point follows the trajectory, the *centre of curvature*, at the corresponding point, describes the *evolute*, and we can take the sign of $d\mathbf{v}$ as *positive* because the sense of its movement follows the *changing sense* of the velocity \mathbf{v} . This result is of the major importance (see the two possible cases in Figs. 1, 2', y 1', 2) because $\frac{d\mathbf{v}}{ds} = dv/ds$ *changes sign*, when the movement is inverted (dv changes to $-dv$ whereas $d\mathbf{v}$, in the *evolute* does not change). When the movement is inverted, the versor $\mathbf{s} \wedge \mathbf{b} = -\mathbf{n}$ *manteins its sense*, but the supplementary acceleration $\mathbf{a}^* = \mathbf{v} \wedge \frac{d\mathbf{v}}{ds} = \mathbf{s} \wedge \mathbf{b} v \frac{dv}{ds}$ *changes it* when $\frac{dv}{ds}$ changes to $-\frac{dv}{ds}$. Consequently, the **reversibility** of the trajectory in CD **does not hold up** in the ND,

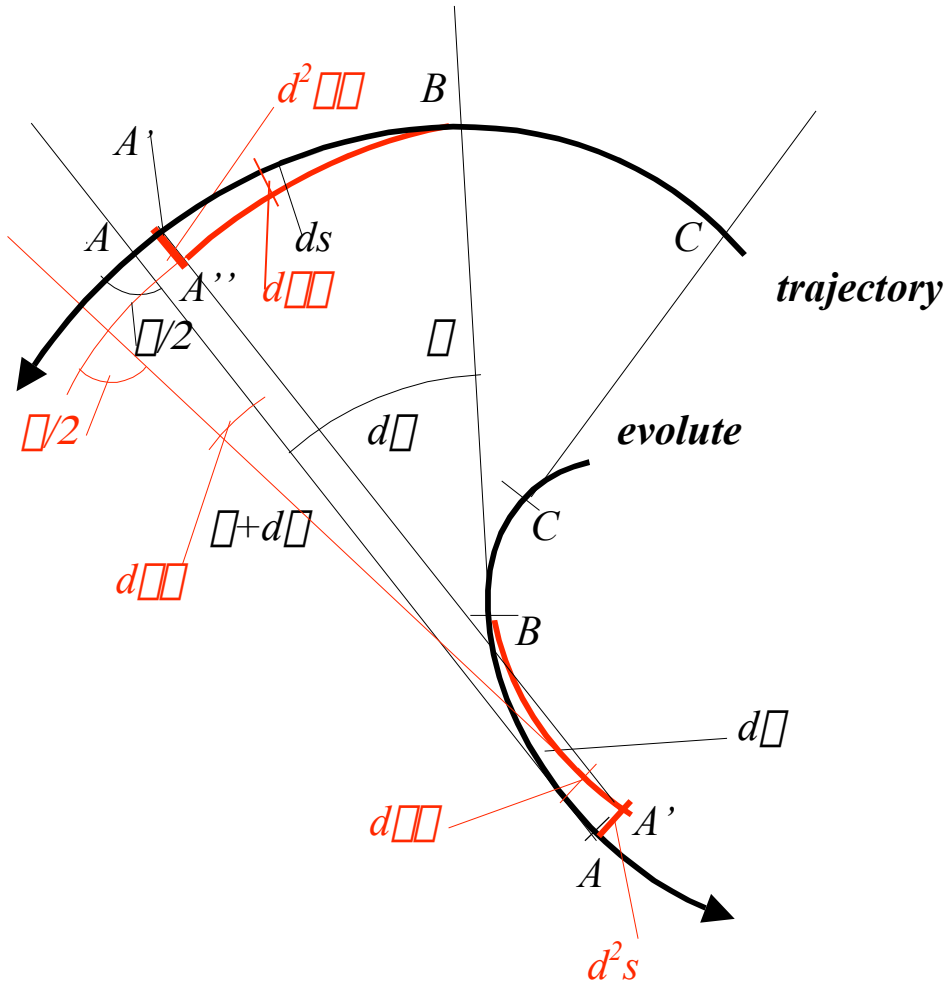
The **CHAOS** presence in physical phenomena has its foundation in this **irreversibility**.



Supplementary Normal Acceleration (when $dv/dt < 0$)

$$a_n^* = d^2 \varphi / dt^2$$

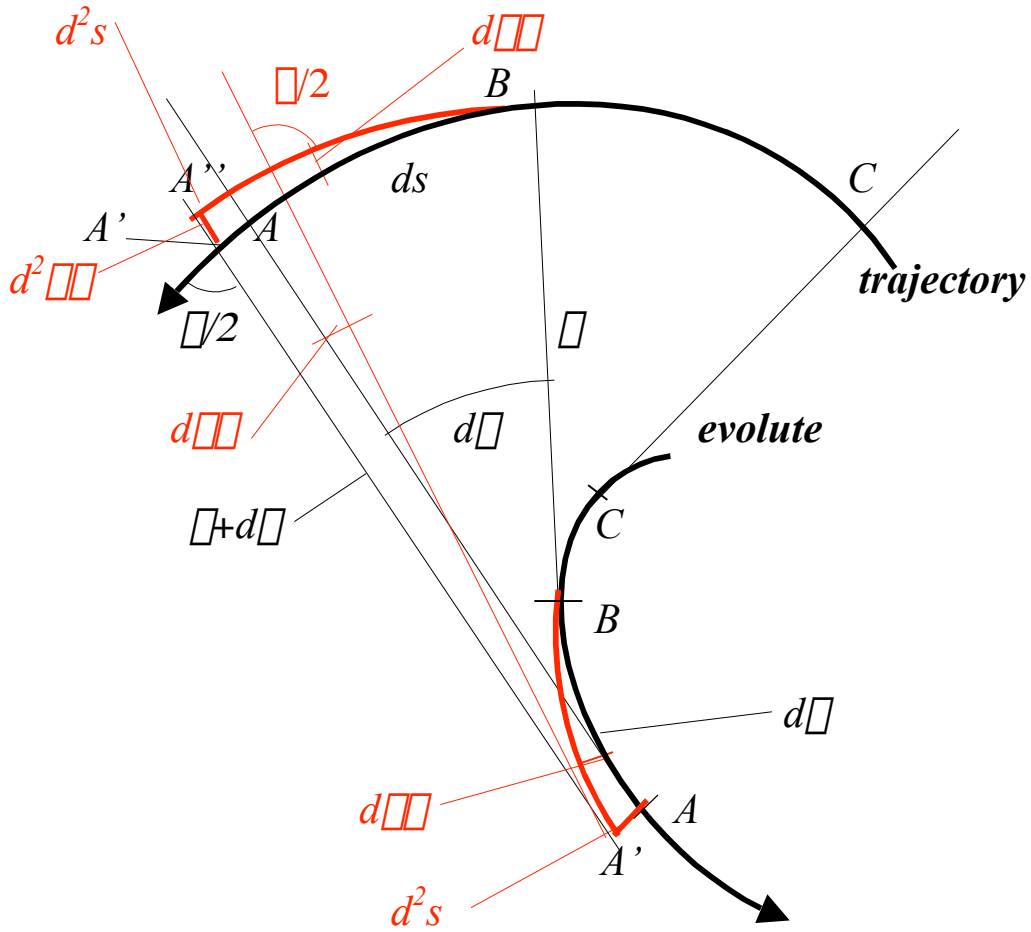
FIG. 2



Supplementary Normal Acceleration
 (running in *inverse direction*, with $dv/dt > 0$)

$$a_n^* = d^2 \phi / dt^2$$

FIG. 1'



Supplementary Normal Acceleration
 (running in *inverse direction*, with $dv/dt < 0$)

$$a_n^* = d^2\phi/dt^2$$

FIG. 2'

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